The estimation of periodic volume increment from permanent inventories with pps-sampling design

or: What is this thing called growth? (Gilbert 1954)

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Overview

• Introduction: terminology & definitions
• Methods
  – pps-Sampling
  – Alternative estimators of growth
  – Model for diameter and height growth
• Application: comparison of different estimators
• Explaining the difference
• Conclusions
A note on terminology

Growth, increment, change
- Growth (rate): increase in size (= increment )
  \[ Y = f(t); \frac{dY}{dt} = f(Y, t) \]
- Forest = population
  - Growth
  - Depletion (loss, drain) due to management (harvest, cut) and natural mortality
  - Ingrowth due to regeneration
  - (area change due to deforestation and/or afforestation)

A note on terminology

Change
- Individual growth = increase in size = positive change
- Forest population aggregate size
  - Net change of growing stock (\(dV = V_2 - V_1\)) can be negative!
Components of growth

Growth and loss (drain) in a period $t_1$ to $t_2$
Subpopulations (domains)
- Survivor trees
- Ingrowth trees
- Loss (trees harvested or naturally died: cut and mortality)

Assumptions
- Two consecutive surveys with a common set of $m$ sample plots
- A pps-sampling design by relascope (Bitterlich ACS) with the same basal area factor
- Diameter (d.b.h.) threshold $d_k$
- Sample trees mapped

Variable of interest: periodic volume increment (“gross annual increment”) per hectare and year
$[m^3ha^{-1}a^{-1}]$
Angle count sampling (ACS):
- Inclusion probability depends on individual basal area $b_k$ and basal area factor $F$: = pps
- **Horwitz-Thompson** is an unbiased estimator:

$$
\hat{Y}_s = \sum_{U_k \in s} \frac{y_k}{\pi_k} = \frac{FA}{b_k}
$$

$$
\hat{Y}_s = FA \sum_{U_k \in s} \frac{y_k}{b_k} = A \sum_{U_k \in s} y_k \frac{F}{b_k} = A \sum_{U_k \in s} y_k w_k
$$

Growth components with repeated pps-sampling

Subsequent survey on permanent plots identifies:
- Survivor trees (re-measured)
- Newly qualified trees
- Cut and mortality trees (measured once at t1)
Growth components with repeated pps-sampling: **survivor trees**

Growth \((y_2 - y_1)\) measurable on repeatedly measured **survivors** only

pps-sampling:

\[
\begin{align*}
    w_{1,k} &= \frac{F}{b_{1,k}}; \\
    w_{2,k} &= \frac{F}{b_{2,k}} \\
    b_{2,k} > b_{1,k} &\Rightarrow w_{2,k} < w_{1,k}
\end{align*}
\]

“Recalibration effect”

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Growth components with repeated pps-sampling: **newly qualified**

**Newly qualified** sample trees measured at subsequent survey only

Regarding the (unobserved) initial diameter \(d_1\)

distinction of two subdomains:

- \(d_1 < d_k\): “true” ingrowth: domain \(I_t\)
- \(d_1 \geq d_k\): ingrowth due to sampling (“nongrowth”):
  actually survivor trees (“recalibration effect”): domain \(S_n\)
Growth components

Increment balance equation with *complete* information

\[ iV = V_{D*} + V_2 - V_1 \]

*\( V_{D*} \): cut and mortality = drain (depletion)\n
*\( V_2 - V_1 \): growing stock net change\n
\[ V_1 = V_{S1} + V_{D1} \]
\[ V_2 = V_{S2} + V_I \]
\[ iV = V_{D*} + V_{S2} + V_I - V_{S1} - V_{D1} \]
\[ iV = \Delta V_D + \Delta V_S + V_I \]

Growth components with repeated pps-sampling

Growth balance equation with *incomplete* information

\[ iV = \Delta V_D + \Delta V_S + V_I \]

1. Component: *growth of depletion trees* (cut & mortality)

\[ \Delta V_D = V_{D*} - V_{D1} \]

Estimator:

\[ \Delta \hat{V}_D = \sum_{U_k \in D} (\hat{v}_{*k} - v_{1,k}) \cdot w_{1,k} \]

Volume at time of removal *\( v_\cdot \) is unknown! → we need a growth model!!
Growth model

Trend function (Sloboda 1971)

\[ y(t_2) = c \cdot \left( \frac{y(t_1)}{c} \right)^{\frac{b}{1-a} \left( t_2^{1-a} - t_1^{1-a} \right)} \]

derived from the differential equation:

\[ \frac{\partial y}{\partial t} = b \cdot \frac{y}{t^a} \cdot \ln \left( \frac{c}{y} \right) \]

\( y = \) diameter or height ; \( t = \) age

Differential equation fitted on survivors

Sloboda trend function: diameter = f(Age) for N. spruce
Growth components with repeated pps-sampling

**Estimator of growth of depletion trees**

\[ \Delta \hat{V}_D = \sum_{U_k \in D} (\hat{v}_{*k} - v_{1,k}) \cdot w_{1,k} \]

Volume at time of removal (= mid of period) \( \hat{v}_* \) estimated based on predicted diameter and height using the Sloboda trend function fitted on the survivor trees observed at \( t_1 \) and \( t_2 \)

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Growth components: **Ingrowth**

\[ i \hat{V} = \hat{\Delta} V_D + \hat{\Delta} V_S + \hat{V}_I \]

\[ \hat{V}_I = \sum_{U_k \in I} v_{2k} \cdot w_{2k} \]

Model-based identification:

- \( i \in I_1 : \hat{d}_{1,i} < d_k \)
- \( i \in S_I : \hat{d}_{1,i} \geq d_k \)

Or based on distances:

- \( i \in I_1 : \text{dist} \leq \frac{d_k}{2 \sqrt{F}} ; F = \text{basal area factor} \)
Growth components: estimators of survivor-growth

\[ i\hat{V} = \hat{\Delta}V_D + \hat{\Delta}V_S + \hat{V}_I \]

(1) Grosenbaugh (1958)
\[ \hat{\Delta}V_S = \sum_{i \in S_r} v_{2,i} \cdot w_{1,i} - \sum_{i \in S_r} v_{1,i} \cdot w_{1,i} \]

(2) Roesch et al. (1989)
\[ \hat{\Delta}V_S = \sum_{i \in S_r} v_{2,i} \cdot w_{2,i} - \sum_{i \in S_r} v_{1,i} \cdot w_{2,i} + \sum_{i \in S_n} v_{2,i} \cdot w_{2,i} - \sum_{i \in S_n} \hat{v}_{1,i} \cdot w_{2,i} \]
\[ \hat{v}_{1,i} = \text{model-based estimated initial volume} \]

(3) Compatible estimator (van Deusen et al. 1986)
\[ \hat{\Delta}V_S = \sum_{i \in S_r} v_{2,i} \cdot w_{2,i} - \sum_{i \in S_r} v_{1,i} \cdot w_{1,i} + \sum_{i \in S_n} v_{2,i} \cdot w_{2,i} \]

The issue of additivity

Net change \( \Delta V \)
\[ \Delta V = V_2 - V_1 \]
\[ iV = V_{D*} + V_2 - V_1 \]
\[ V_2 - V_1 = iV - V_{D*} \]
\[ i\hat{V} = \hat{\Delta}V_S + \hat{\Delta}V_D + \hat{V}_I \]
\[ \hat{\Delta}V = i\hat{V} - \hat{V}_{D*} \]
\[ \hat{\Delta}V = \sum_{i \in S_r \cup i \in S_n} v_{2,i} \cdot w_{2,i} - \sum_{i \in S_r} v_{1,i} \cdot w_{1,i} - \sum_{i \in D} v_{1,i} \cdot w_{1,i} \]
\[ + \sum_{i \in I_n} v_{2,i} \cdot w_{2,i} \]

Compatible survivor-growth estimator
Application: German NFI in Baden-Württemberg

Estimation of **periodic annual volume increment** per hectare (= gross annual increment G.A.I.) 2002-2012

Estimation is based on survivor-growth estimator according to Roesch et al. (1989)

<table>
<thead>
<tr>
<th>Net change of ...</th>
<th>Survivors</th>
<th>Depletion</th>
<th>Ingrowth</th>
<th>G.A.I. [m³ ha⁻¹ yr⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.25</td>
<td>1.73</td>
<td>0.31</td>
<td>12.29 ± 0.08</td>
</tr>
<tr>
<td></td>
<td>83.4%</td>
<td>14.0%</td>
<td>2.5%</td>
<td></td>
</tr>
</tbody>
</table>

**Gross annual increment**

**Comparison** of estimators differing with regard to survivor-growth component

<table>
<thead>
<tr>
<th>Estimator</th>
<th>G.A.I [m³ ha⁻¹ a⁻¹]</th>
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<td>Grosenbaugh</td>
<td>12.16</td>
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<td>Roesch et al.</td>
<td>12.29 ± 0.08</td>
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<td>van Deusen</td>
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Striking difference!
Explaining the differences

Hypothesis 1
Incomplete sampling at initial survey (overlooked trees): initial growing stock underestimated

Correction: identification of newly qualified trees which might have been overlooked at $t_1$:
a part of newly qualified trees ($S_N$) is re-classified as “to-be-repeatedly-measured” survivor trees based on their distances and estimated initial diameters (probably overlooked trees)
(2 variants A and B regarding proportion of reclassified $S_N$ trees)

Correction of initial growing stock

<table>
<thead>
<tr>
<th>Estimator</th>
<th>G.A.I [m³ ha⁻¹ a⁻¹]</th>
<th>Original</th>
<th>with correction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Grosenbaugh</td>
<td>12.16</td>
<td>12.33</td>
<td>12.51</td>
</tr>
<tr>
<td>Roesch et al.</td>
<td>12.29</td>
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<tr>
<td>van Deusen</td>
<td>13.21</td>
<td>12.83</td>
<td>12.54</td>
</tr>
</tbody>
</table>
Correction of initial growing stock: explanation

Initial volume increases as a subset of survivors newly qualified at $t_2$ (“non-growth”) are now added to initial survivors (with modelled volume and weights calculated from the modelled initial diameters)

Effect on Grosenbaugh: survivor growth increases

Effect on van Deusen: difference between $V_2$ and $V_1$ decreases:

No effect on Roesch et al. (re-classification does not affect survivor growth estimate)

Explaining the differences

Hypothesis 2

Diameter growth is underestimated by the model (trend function)

Correction:

Initial diameters of newly qualified survivors (“backwards” predicted by the growth model) are increased (restricted to trees with a diameter at $t_2 \leq 30$ cm)
Correction of initial growing stock **and diameter growth**

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Correction of diameter growth: explanation

Significant effect on Roesch et al. estimator of survivor growth as it depends on modelled growth of newly qualified survivors

Minor effect on Grosenbaugh- and van Deusen-estimator: due to increased ingrowth volume as ingrowth trees are determined by estimated initial diameters: increased diameter growth leads to a higher proportion of trees with initial diameter below diameter threshold \(d_k\)
Conclusions

• **pps-sampling**
  – Inclusion probabilities of repeatedly measured trees change and newly qualified trees appear at subsequent survey (recalibration effect -> inflation of variance)
  – … allows different unbiased estimators of survivor growth

• **Growth** is a **susceptible** variable: difference of large quantities; strongly depends on correct (complete) sampling; however, … no survey is perfect!

• Estimator depending on model may be affected by model bias

Conclusions

• **Additivity** (“compatibility”) depends on survivor-growth estimator used (but, is additivity necessary?)

• German NFI: “Roesch”-estimator without correction for additivity:
  – Pro: independent of initial measurements, mainly relies on current \( t_2 \) survey
  – Contra:
    • depends on (unbiased) diameter growth model
    • not compatible (additive) with net volume change