The estimation of periodic volume increment from permanent inventories with pps-sampling design

or: What is this thing called growth? (Gilbert 1954)

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Overview

- Introduction: terminology & definitions
- Methods
  - pps-Sampling
  - Alternative estimators of growth
  - Model for diameter and height growth
- Application: comparison of different estimators
- Explaining the difference
- Conclusions

A note on terminology

Growth, increment, change

- Growth (rate): increase in size (= increment)
  Y = f(t); dY/dt =f(Y, t)
- Forest = population
  - Growth
  - Depletion (loss, drain) due to management (harvest, cut) and natural mortality
  - Ingrowth due to regeneration
  - (area change due to deforestation and/or afforestation)

A note on terminology

Change

- Individual growth = increase in size = positive change
- Forest population aggregate size
  - Net change of growing stock ($dV = V_2 – V_1$) can be negative!

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Components of growth

Growth and loss (drain) in a period t_1 to t_2

Subpopulations (domains)

- Survivor trees
- Ingrowth trees
- Loss (trees harvested or naturally died: cut and mortality)

Assumptions

- Two consecutive surveys with a common set of *m* sample plots
- A pps-sampling design by relascope (Bitterlich ACS) with the same basal area factor
- Diameter (d.b.h.) threshold d_k
- Sample trees mapped

  … at both occasions.

Variable of interest: periodic volume increment ("gross annual increment") per hectare and year [$m^3 ha^{-1} a^{-1}$]

pps-sampling

Angle count sampling (ACS):

- Inclusion probability depends on individual basal area b_k and basal area factor *F*: = pps
- **Horwitz-Thompson** is an unbiased estimator:

$$\hat{Y}_s = \sum_{U_k \in s} \frac{y_k}{\pi_k}; \pi_k = \frac{FA}{b_k}$$

$$\hat{Y}_s = FA \sum_{U_k \in s} \frac{y_k}{b_k} = A \sum_{U_k \in s} y_k \frac{F}{b_k} = A \sum_{U_k \in s} y_k w_k$$

Growth components with repeated pps-sampling

Subsequent survey on permanent plots identifies:

- Survivor trees (re-measured)
- Newly qualified trees
- Cut and mortality trees (measured once at t1)

Growth components with repeated pps-sampling: **survivor trees**

Growth ($y_2 - y_1$) measurable on repeatedly measured **survivors** only

pps-sampling:

$$w_{1.k} = \frac{F}{b_{1.k}}; w_{2.k} = \frac{F}{b_{2.k}}$$

$$b_{2.k} > b_{1.k} \Rightarrow w_{2.k} < w_{1.k}$$

“Recalibration effect”

Growth components with repeated pps-sampling: **newly qualified**

Newly qualified sample trees measured at subsequent survey only

Regarding the (unobserved) initial diameter d_1 distinction of two subdomains:

- $d_1 < d_k$: “true” ingrowth: domain $\boldsymbol{I_t}$
- $d_1 \geq d_k$: ingrowth due to sampling (“nongrowth”): actually survivor trees (“recalibration effect”): domain $\boldsymbol{S_n}$

Growth components

Increment balance equation with *complete* information

$$iV = V_{D*} + V_2 - V_1$$

$$V_{D*} : \text{cut and mortality} = \text{drain (depletion)}$$

$$V_2 - V_1 : \text{growing stock net change}$$

$$V_1 = V_{S1} + V_{D1}$$

$$V_2 = V_{S2} + V_I$$

$$iV = V_{D*} + V_{S2} + V_I - V_{S1} - V_{D1}$$

$$iV = \Delta V_D + \Delta V_S + V_I$$

Growth components with repeated pps-sampling

Growth balance equation with *incomplete* information

$$iV = \Delta V_D + \Delta V_S + V_I$$

1. Component: **growth of depletion trees** (cut & mortality)

$$\Delta V_D = V_{D*} - V_{D1}$$

Estimator:

$$\Delta \hat{V}_D = \sum_{U_k \in D} (\hat{v}_{*k} - v_{1.k}) \cdot w_{1.k}$$

Volume at time of removal ***v*** $_*$ is unknown!

→ we need a growth model!

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Growth model

Trend function (Sloboda 1971)

$$y_{(t_2)} = c \cdot \left(\frac{y_{(t_1)}}{c} \right)^{e^{-\frac{b}{1-a} \cdot \left(t_2^{(1-a)} - t_1^{(1-a)} \right)}}$$

derived from the differential equation:

$$\frac{\partial y}{\partial t} = b \cdot \frac{y}{t^a} \cdot \ln\left(\frac{c}{y} \right)$$

y = diameter or height ; *t* = age

Differential equation fitted on survivors

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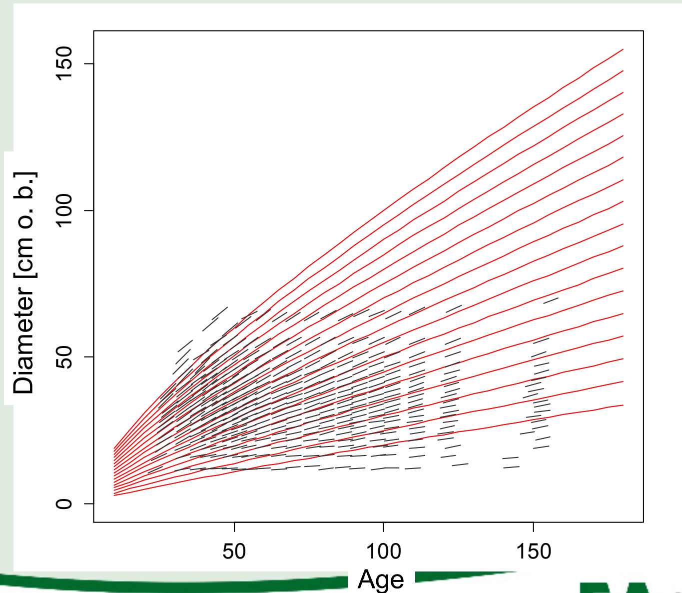
Sloboda trend function: diameter = f(Age) for N. spruce



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Growth components with repeated pps-sampling

Estimator of growth of depletion trees

$$\Delta\hat{V}_D = \sum_{U_k \in D} (\hat{v}_{*k} - v_{1.k}) \cdot w_{1.k}$$

Volume at time of removal (= mid of period) $\hat{v}_*$ estimated based on predicted diameter and height using the Sloboda trend function fitted on the survivor trees observed at t_1 and t_2

Growth components: **Ingrowth**

$$i\hat{V} = \hat{\Delta}V_D + \hat{\Delta}V_S + \hat{V}_I$$

$$\hat{V}_I = \sum_{U_k \in I} v_{2k} \cdot w_{2k}$$

Model-based identification:

$$i \in I_t : \hat{d}_{1.i} < d_k$$

$$i \in S_I : \hat{d}_{1.i} \geq d_k$$

Or based on distances:

$$i \in I_t : dist \leq \frac{d_k}{2\sqrt{F}}; F = \text{basal area factor}$$

Growth components: estimators of **survivor-growth**

$$i\hat{V} = \hat{\Delta}V_D + \boxed{\hat{\Delta}V_S} + \hat{V}_I$$

(1) Grosenbaugh (1958)

$$\hat{\Delta}V_S = \sum_{i\in S_r} v_{2.i} \cdot w_{1.i} - \sum_{i\in S_r} v_{1.i} \cdot w_{1.i}$$

(2) Roesch et al. (1989)

$$\hat{\Delta}V_S = \sum_{i\in S_r} v_{2.i} \cdot w_{2.i} - \sum_{i\in S_r} v_{1.i} \cdot w_{2.i} + \sum_{i\in S_n} v_{2.i} \cdot w_{2.i} - \sum_{i\in S_n} \hat{v}_{1.i} \cdot w_{2.i}$$

$$\hat{v}_{1.i} = \text{model - based estimated initial volume}$$

(3) Compatible estimator (van Deusen et al. 1986)

$$\hat{\Delta}V_S = \sum_{i\in S_r} v_{2.i} \cdot w_{2.i} - \sum_{i\in S_r} v_{1.i} \cdot w_{1.i} + \sum_{i\in S_n} v_{2.i} \cdot w_{2.i}$$

The issue of **additivity**

Net change *ΔV*

$$\Delta V = V_2 - V_1$$

$$iV = V_{D*} + V_2 - V_1$$

$$V_2 - V_1 = iV - V_{D*}$$

$$i\hat{V} = \hat{\Delta}V_S + \hat{\Delta}V_D + \hat{V}_I$$

$$\Delta\hat{V} = i\hat{V} - \hat{V}_{D*}$$

Compatible survivor-growth estimator

$$\Delta\hat{V} = \boxed{\sum_{i\in S_r \cup i\in S_n} v_{2.i} \cdot w_{2.i} - \sum_{i\in S_r} v_{1.i} \cdot w_{1.i}} - \sum_{i\in D} v_{1.i} \cdot w_{1.i} + \sum_{i\in I_n} v_{2.i} \cdot w_{2.i}$$

Application: German NFI in Baden-Württemberg

Estimation of **periodic annual volume increment** per hectare (= gross annual increment G.A.I.) 2002-2012

Estimation is based on survivor-growth estimator according to Roesch et al. (1989)

Net change of …				
Survivors	Depletion	Ingrowth	**G.A.I.**	
[m³ ha^{-1} yr^{-1}]				
10.25	1.73	0.31	**12.29**	**± 0.08**
83.4%	14.0%	2.5%		

Gross annual increment

Comparison of estimators differing with regard to survivor-growth component

Estimator	G.A.I [m³ ha^{-1} a^{-1}]
Grosenbaugh	12.16
Roesch et al.	12.29 ± 0.08
van Deusen	13.21

Striking difference!

Explaining the differences

Hypothesis 1

Incomplete sampling at initial survey (overlooked trees): initial growing stock underestimated

Correction: identification of newly qualified trees which might have been overlooked at t_1:
a part of newly qualified trees (S_N) is re-classified as "to-be-repeatedly-measured" survivor trees based on their distances and estimated initial diameters (probably overlooked trees)
(2 variants A and B regarding proportion of reclassified S_N trees)

Correction of initial growing stock

Estimator	G.A.I [$m^3\ ha^{-1}\ a^{-1}$]		
	original	with correction	
		A	B
Grosenbaugh	12.16	12.33	12.51
Roesch et al.	12.29	12.29	12.29
van Deusen	13.21	12.83	12.54

Correction of initial growing stock: explanation

Initial volume increases as a subset of survivors newly qualified at t_2 ("non-growth") are now added to initial survivors (with modelled volume and weights calculated from the modelled initial diameters)

Effect on Grosenbaugh: survivor growth increases

Effect on van Deusen: difference between V_2 and V_1 decreases:

No effect on Roesch et al. (re-classification does not affect survivor growth estimate)

Explaining the differences

Hypothesis 2

Diameter growth is underestimated by the model (trend function)

Correction:

Initial diameters of newly qualified survivors ("backwards" predicted by the growth model) are increased (restricted to trees with a diameter at t_2 ≤ 30 cm)

Correction of initial growing stock **and diameter growth**

Estimator	G.A.I [m^3 ha^{-1} a^{-1}]		
	original	with correction	
		A	B
Grosenbaugh	12.16	12.43	12.56
Roesch et al.	12.29	12.62	12.62
van Deusen	13.21	12.90	12.65

Correction of diameter growth: explanation

Significant effect on Roesch et al. estimator of survivor growth as it depends on modelled growth of newly qualified survivors

Minor effect on Grosenbaugh- and van Deusen-estimator: due to increased ingrowth volume as ingrowth trees are determined by estimated initial diameters: increased diameter growth leads to a higher proportion of trees with initial diameter below diameter threshold d_k

Conclusions

- pps-sampling
  - Inclusion probabilities of repeatedly measured trees change and newly qualified trees appear at subsequent survey (**recalibration** effect -> inflation of variance)
  - … allows different unbiased estimators of survivor growth
- Growth is a **susceptible** variable: difference of large quantities; strongly depends on correct (complete) sampling; however, … no survey is perfect!
- Estimator depending on model may be affected by model bias

Conclusions

- **Additivity** ("compatibility") depends on survivor-growth estimator used (but, is additivity necessary?)
- German NFI: "Roesch"-estimator without correction for additivity:
  - Pro: independent of initial measurements, mainly relies on current (t_2) survey
  - Contra:
    - depends on (unbiased) diameter growth model
    - not compatible (additive) with net volume change