Local density

Definition and basic properties

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USEWOOD WG2, Training school in Dublin, 16.-19. September 2013

Contents

Definition of local density

Local density has been introduced by Mandallaz [1991, section 3.1 starting on page 18]. A key property of local density is defined by

$$\int_D Y(x)dx = Y. \tag{1}$$

The definite integral of local density $Y(x)$ in domain $D \subset \mathbb{R}^2$ (geographical area) equals the total amount of variable Y in D.

Expected value of local density equals the mean value of Y in D

$$\mathbb{E}\big[Y(x)\big] = \frac{1}{\lambda(D)} \int_D Y(x)dx = \bar{Y}, \tag{2}$$

where $\lambda(D)$ stands for the total area of D.

The utility of local density I

Naturalness

Parameter estimation proceeds by **continuous-population** approach **avoiding imperfections of finite population methodology when applied to NFIs**.

- study area D can't be tessellated into congruent units (population elements) of circular (or any other) shape of sample plots[a]
- unknown size of some populations e. g. trees.
- fuzziness of the definition of population's elements[b] e. g. dead wood, streams, roads . . .

[a] Sample plots are not population's elements!
[b] Field work complications.

The utility of local density II

Simplicity of estimation procedures

After the introduction of local density, all parameters can be estimated using an unified approach - **the Horwitz-Thompson theorem for continuous populations** [Cordy, 1993].

Flexibility

Sampling design and protocols can be defined in such a way that for any variable **we can evaluate local density anywhere in** D.

Point sampling

In the very typical and simplest case, point sampling is used **to estimate the extent of arbitrarily defined subregions[1] within the survey area or geographical domain D.**

Local density is defined as an **indicator variable** expressing a membership of inventory (sample) point x to particular land-category K

$$I_K(x) = \begin{cases} 1 & \text{if } x \in K \\ 0 & \text{if } x \notin K. \end{cases} \tag{3}$$

[1] For example the total area of a land-use category.

Following description was borrowed from Mandallaz [2007, section 4.2 starting on page 55].

Let's define **concentric circles** K_r (one, usually two or three) on each inventory (sample) point.
Stem i **is selected if and only if it is located inside a circle** $K_r(x) = \{y \in \mathbb{R}^2 \mid d(x, y) \leq r\}$ **with radius** r **and center point coincident with the inventory point** x**.**

The notation $d(x, y)$ is used for Euclidean distance between x and y (an arbitrary point within the circle). Position of a stem is given by orthogonal projection of a point lying on the central axis in breast height (1.3 m) into the 2D plane of the coordinate system in use.

Selection of stems on circular plots II

Now let's define an indicator variable $I_i(x)$ for stem i and inventory point x as

$$I_i(x) = \begin{cases} 1 & \text{if } u_i \in K_r(x) \\ 0 & \text{if } u_i \notin K_r(x). \end{cases} \tag{4}$$

Furthermore define N **circles** $K_i(r) = K_r(u_i)$ **with constant radius** r **and center points** u_i **coincident with stem positions**. **Stem** i **gets selected if an only if any of the inventory points** x **is falls into circle** $K_i(r)$:

$$I_i(x) = 1 \Leftrightarrow x \in K_i(r). \tag{5}$$

The circle $K_i(r)$ is called **inclusion zone** of the stem i. To each stem i we assign an inclusion zone the size of which in the most typical case depends on particular stem's attributes (dbh, height, species etc.).

Selection of stems on circular plots III

Definition of local density, sampling protocol for stems

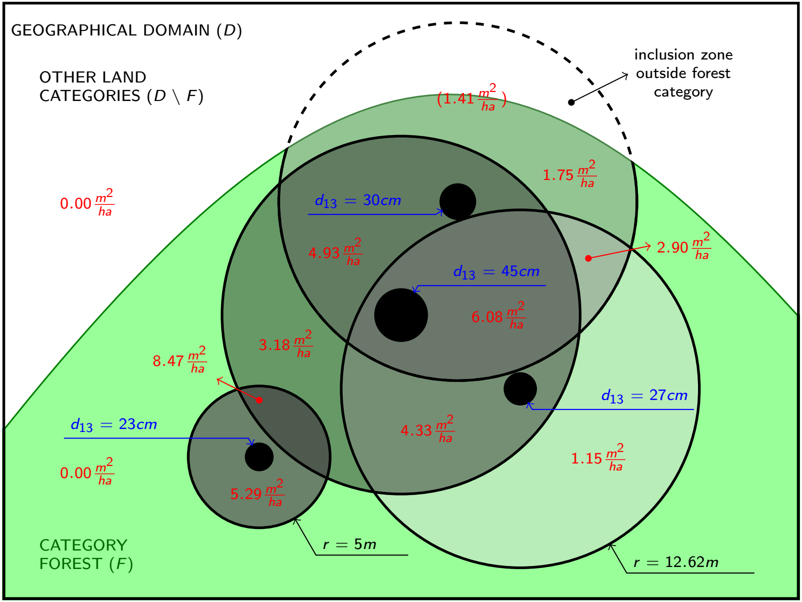
Local density function corresponding to above described sampling protocol is given by

$$Y(x) = \sum_{i \in \mathcal{P}} \frac{I_i(x) Y_i}{\lambda(K_i \cap F \cap A)}. \quad (6)$$

- $\lambda(K_i)$ is the size of the circular segment K_i for stem i
- $\mathcal{P}$ is a finite population of stems located on accessible areas A within geographical domain D and within forest landuse-category F
- Y_i is the observed quantity on stem i we want to estimate in D
- $Y(x)$ follows (1) and (2)
- $\lambda(K_i \cap F \cap A)$ is the common area of regions K_i, F and A

Definition 6 supposes that plots with center point outside forest or inside inaccessible area are never established. The area in the denominator compensates for the edge-effect, which would otherwise occur.

Local density, selection of stems, CZNFI2



Modifications of local density I

Relascope (or Bitterlich) sampling

- also known as **the angle count technique**
- highly effective sampling protocol for stems
- see Mandallaz [2007, section 4.2 starting on page 57] and Gregoire and Valentine [2008, section 8 starting on page 247] for details

Line intersect sampling

- very common extensions are **various protocols using transects**[a]
- original reference is Kaiser [1983]
- see Gregoire and Valentine [2008, section 9 starting on page 270] for a thorough and useful introduction

[a] Estimating total length of linear objects, total area of aerial objects, total volume of 3D objects e. g. lying dead wood.

Modifications of local density II

Generalized local density

- local density modified for **two-stage selection procedure** at the sample plot level
- **some stem-specific variables (e. g. height and upper diameter) are measured on a subset of stems registered on the plot**
- see Mandallaz [2007, section 4.4 starting on page 69]

Tract sampling - reduction of travel cost

- tracts are defined by a fixed number of **translation vectors**
- each of these vectors defines position of one inventory plot which altogether form a tract (e. g. plots organized in a square tract)
- see Mandallaz [2007, section 4.3 starting on page 65] and Gregoire and Valentine [2008, section 7 starting on page 207]

Modifications of local density III

Edge effect corrections

- for specific sampling protocols an **edge effect** occurs
- **it leads to underestimation** of the amount of resource in D
- in most cases **the definition of local density has to be adjusted to compensate for the edge effect**
- see Mandallaz [2007, section 4.2 starting on page 52] and Gregoire and Valentine [2008, section 7.5 starting on page 223] for a complete treatment of the topic

Literature

C. B. Cordy. An extension of the horwitz-thompson theorem to point sampling from a continuous universe. *Statistics and Probability Letters*, 18:353–362, 1993.

T. G. Gregoire and H. T. Valentine. *Sampling Strategies For Natural Resources and The Environment*. Chapman and Hall/CRC, 2008.

L. Kaiser. Unbiased estimation in line-intercept sampling. *Biometrics*, 39: 965–976, 1983.

D. Mandallaz. *An unified approach to sampling theory for forest inventory based on infinite population and superpopulation models*. PhD thesis, Swiss Federal Institute of Technology (ETH), Zurich, 1991.

D. Mandallaz. *Sampling Techniques For Forest Inventories*. Chapman and Hall/CRC, 2007.